1. Bob stands 15 miles west of Alice. Alice starts moving west at 3 mph, and Bob starts moving east at 6 mph. They stop moving once they meet. How far is Bob from his starting position?

   Answer: $\boxed{\mathbf{10}}$

   Solution: Let b miles be how far Bob is from his starting position when he meets Alice. Bob is moving at twice the speed as Alice, so when they meet, Bob should have moved twice the distance as Alice. Thus, Alice should be a distance $\frac{b}{2}$ miles from her starting position when she meets Bob. When the two meet, they should have moved a combined distance of 15 miles. Solving $b + \frac{b}{2} = 15$, we get that $b = \boxed{\mathbf{10}}$

2. A perfect square is a number that can be represented as the square of a positive integer. How many positive perfect squares less than 1000 are there?

   Answer: $\boxed{\mathbf{31}}$

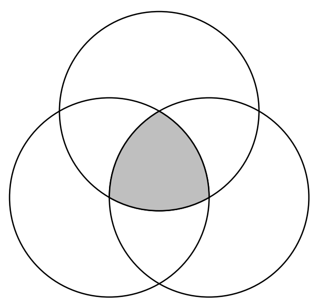
   Solution: The greatest perfect square less than 1000 will be a number of the form n^2 such that $n^2 < 1000 \leq (n+1)^2$. To find the greatest perfect square, we can quickly find that $30^2 = 900$, so we can guess-and-check numbers slightly more than 30. We find that $31^2 = 961$ and $32^2 = 1024$, so 31^2 is the the largest perfect square less than 1000. Since $1^2, 2^2, \ldots, 30^2, 31^2$ are all less than 1000, there must be $\boxed{\mathbf{31}}$ positive perfect squares less than 1000.

3. If 5 and 8 are solutions to the quadratic equation $x^2 + ax + b = 0$, what is $b - a$?

   Answer: $\boxed{\mathbf{53}}$

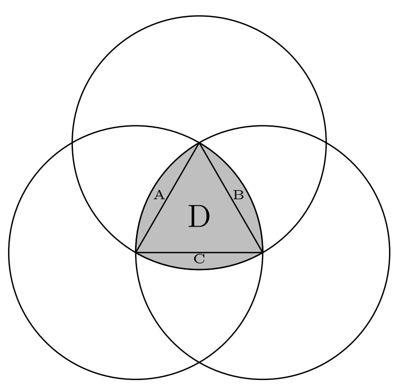
   Solution: By Vieta's formulas, we have that $a = -(5+8) = -13$ and $b = 5 \cdot 8 = 40$, so $b - a = \boxed{\mathbf{53}}$.

4. Consider three circles of radii 2 positioned such that every circle passes through the center of the other two circles. What is the area of the region common to all three circles?



   Answer: $\boxed{\mathbf{2\pi - 2\sqrt{3}}}$

   Solution: Denote the centers of the circles as O_1, O_2, and O_3. Note that these centers all equidistant to each other, so $\angle O_1O_2O_3 = \angle O_2O_3O_1 = \angle O_3O_1O_2 = 60^\circ$. We label sub regions of the common region as A, B, C, and D, as shown below:



Note that because each of the regions $A+D$, $B+D$, and $C+D$ are arcs with angle $60°$, $A+D = B+D = C+D = \frac{60°}{360°}(\pi r^2) = \frac{1}{6}\pi r^2 = \frac{2\pi}{3}$. Thus, $A+B+C+3D = 2\pi$.

Additionally, we have that D is an equilateral triangle with side lengths lying on the radii of the circles, so it has side length 2. Thus, the area of D can be found with the formula $A = \frac{\sqrt{3}}{4}r^2$, which is $\sqrt{3}$ for $r = 2$. The total area of the shaded region is $A+B+C+D = A+B+C+3D-2D = \boxed{\mathbf{2\pi - 2\sqrt{3}}}$.

5. A prime number a less than 12 and a positive perfect square b less than 16 are selected uniformly and independently at random. What is the probability that $a+b$ is also a perfect square? Express your answer as a fraction in lowest terms.

   Answer: $\boxed{\mathbf{\frac{1}{5}}}$

   Solution: We have that $a \in \{2, 3, 5, 7, 11\}$ and $b \in \{1, 4, 9\}$, meaning that there are $5 \cdot 3 = 15$ possible pairs for (a, b). Going through every possible pair, we find that there are 3 pairs (a, b) where $a+b$ is a perfect square: $(3, 1)$, $(5, 4)$, and $(7, 9)$. Thus, the probability that $a+b$ is a perfect square is $\frac{3}{15} = \boxed{\mathbf{\frac{1}{5}}}$.

6. What is the area of the largest triangle that can be inscribed in a semicircle of radius 5?

   Answer: $\boxed{\mathbf{25}}$

   The largest triangle inscribed in a semicircle will be an isosceles right triangle, where the hypotenuse is completely on the diameter of the semicircle. This triangle has a base of $2r$ (the diameter) and a height of r (the radius), where r is the radius of the semicircle, thus the area of this triangle is r^2. Setting $r = 5$, we find that the area is $\boxed{\mathbf{25}}$.

7. How many positive integer factors does the number $(1+3+5+\cdots+97+99)^2$ have?

   Answer: $\boxed{\mathbf{45}}$

   Solution: Using the formula for an arithmetic series, we have that:

$$\begin{aligned} 1+3+5+\ldots+97+99 &= 50\left(\frac{1+99}{2}\right) \\ &= 50 \cdot 50 \\ &= 50^2 \end{aligned}$$

   Thus, $(1+3+5+...+97+99)^2 = 50^4 = 5^8 \cdot 2^4$. The number of factors of $5^8 \cdot 2^4$ is $(8+1)(4+1) = \boxed{\mathbf{45}}$

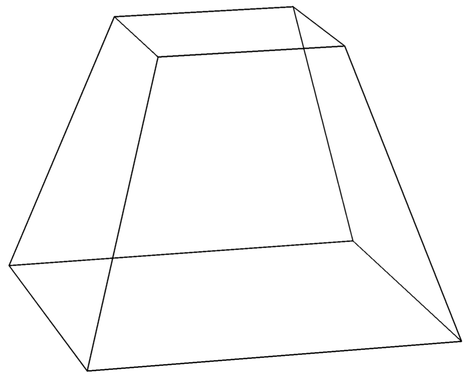
8. The Nueva Upper School campus has three floors. During a school day, each student has 4 classes. Kyle always starts off the day with a class on the first floor. Each of his next 3 classes is on a random floor, chosen with equal probability. Before school, his classmate Riley peeks at Kyle's schedule and tells Kyle that none of his back-to-back classes are on the same floor that day. What is the probability that his fourth class is on the first floor? Express your answer as a fraction in the lowest terms.

   Answer: $\boxed{\mathbf{\frac{1}{4}}}$

   Solution: There are 2 possible floors for the second class (all floors except the first class's floor), 2 possible floors for the third class (all floors except the second class's floor), and 2 possible floors for the fourth class (all floors except the third class's floor) . Thus, there are $2 \cdot 2 \cdot 2 = 8$ possibilities for Kyle's schedule.

   In order for the fourth class to be on the first floor, the third class needs to not be on the first floor. Thus, there are 2 possible floors for the second class, 1 possible floor for the third class (the floor that is not the second class's floor or the first floor), and 1 possible floor for the fourth class (the first floor). Thus, the probability that Kyle's class is on the first floor is $\frac{2}{8} = \boxed{\mathbf{\frac{1}{4}}}$

9. Find the volume of a truncated pyramid with a square base and a side length of 10 cm, a square top with a side length of 5 cm, and a height of 20 cm.



Answer: $\boxed{\frac{3500}{3}}$

Solution: This truncated pyramid can be thought of a smaller pyramid "sliced" off of a larger pyramid. The larger pyramid has a square base with side length 10 cm and a height of 40 cm. The smaller pyramid has a square base with side length 5 cm and a height of 20 cm. The volume of the larger pyramid is $\frac{1}{3}(10^2)(40) = \frac{4000}{3}$ cm^3 and the volume of the smaller pyramid is $\frac{1}{3}(5^2)(20) = \frac{500}{3}$ cm^3, so the volume of the truncated pyramid is $\frac{4000}{3} - \frac{500}{3} = \boxed{\frac{3500}{3}}$.

10. Let x be a real number satisfying the equation

$$27 \cdot 4^x + 8 \cdot 9^x = 30 \cdot 6^x.$$

Find the sum of all possible values of x.

Answer: $\boxed{3}$

Solution: If we divide by 4^x on both sides of the equation, we get:

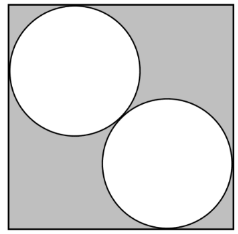
$$27 \cdot \frac{4^x}{4^x} + 8 \cdot \frac{9^x}{4^x} = 30 \cdot \frac{6^x}{4^x}$$
$$27 + 8 \cdot \left(\frac{3^x}{2^x}\right)^2 = 30 \cdot \left(\frac{3^x}{2^x}\right)$$
$$8 \cdot \left(\frac{3^x}{2^x}\right)^2 - 30 \cdot \left(\frac{3^x}{2^x}\right) + 27 = 0$$

Substituting $y = \frac{3^x}{2^x}$, we have:

$$8y^2 - 30y + 27 = 0$$

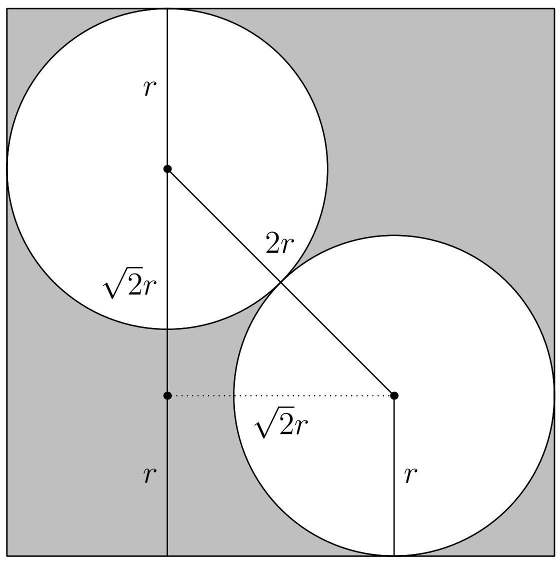
Using the quadratic formula, we find that $y = \dfrac{30 \pm \sqrt{900 - 864}}{16} = \dfrac{3}{2}, \dfrac{9}{4}$. Thus, $x = 1, 2$, so the sum of all possible values of x is $\boxed{3}$.

11. Two identical circles are tangent to a square of side length 1 and to each other as shown in the diagram. If the area of the shaded region can be expressed as $a - b\pi + c\sqrt{2}\pi$, where a, b, and c are positive integers, compute the value of $a + b + c$.



Answer: $\boxed{\mathbf{6}}$

Solution: To find the area of the shaded region, we find the area of the two circles and subtract it from the area of the square. Let r be the radius of each circle. Consider the diagram below:



Based on this diagram, we find that the side length of the square can be expressed as $(2+\sqrt{2})r$. Setting this value equal to 1 gives $r = \dfrac{1}{2+\sqrt{2}} = \dfrac{2-\sqrt{2}}{2}$.

The combined area of these two circles is $2\pi r^2 = \pi(3-2\sqrt{2})$. Subtracting this area from the square, we get that the area of the shaded region is $1-3\pi+2\pi\sqrt{2}$. Thus, the answer is $1+3+2=\boxed{\mathbf{6}}$.

12. Compute the infinite sum
$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} = \frac{1^2}{2^1} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots.$$

Answer: $\boxed{\mathbf{6}}$

Solution: Let the desired sum be S. Then:

$$\begin{aligned}
S &= 2S - S \\
&= \left(\frac{1^2}{2^0}+\frac{2^2}{2^1}+\frac{3^2}{2^2}+\frac{4^2}{2^3}+\cdots\right)-\left(\frac{1^2}{2^1}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\frac{4^2}{2^4}+\cdots\right) \\
&= \frac{1^2-0^2}{2^0}+\frac{2^2-1^2}{2^1}+\frac{3^2-2^2}{2^2}+\frac{4^2-3^2}{2^3}+\cdots \\
&= \frac{(1-0)(1+0)}{2^0}+\frac{(2-1)(2+1)}{2^1}+\frac{(3-2)(3+2)}{2^2}+\frac{(4-3)(4+3)}{2^3}+\cdots \\
&= \frac{1}{2^0}+\frac{3}{2^1}+\frac{5}{2^2}+\frac{7}{2^3}+\cdots.
\end{aligned}$$

Repeating the above and applying the geometric series summation formula yields:

$$\begin{aligned}
S &= 2S - S \\
&= \left(2+\frac{3}{2^0}+\frac{5}{2^1}+\frac{7}{2^2}+\cdots\right)-\left(\frac{1}{2^0}+\frac{3}{2^1}+\frac{5}{2^2}+\frac{7}{2^3}+\cdots\right) \\
&= 2+\frac{2}{2^0}+\frac{2}{2^1}+\frac{2}{2^2}+\cdots \\
&= 2+\frac{2}{1-\frac{1}{2}} \\
&= \boxed{\mathbf{6}}.
\end{aligned}$$

13. What are the last two digits of $41^{42^{18^{9^{5^3}}}}$?

Answer: $\boxed{\mathbf{41}}$

Solution: We can find that:

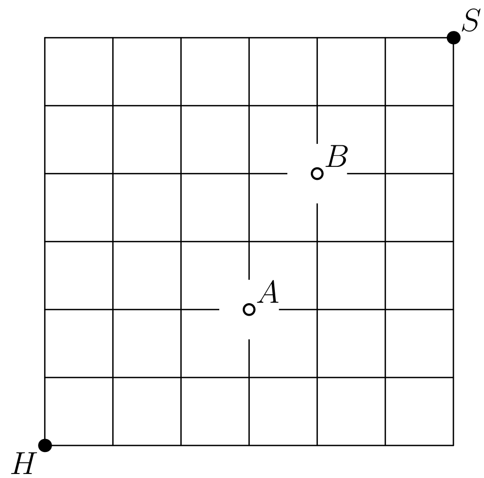
$$\begin{aligned}
41^1 &\equiv 41 \mod 100 \\
41^2 &\equiv 81 \mod 100 \\
41^3 &\equiv 21 \mod 100 \\
41^4 &\equiv 61 \mod 100 \\
41^5 &\equiv 01 \mod 100 \\
41^6 &\equiv 41 \mod 100
\end{aligned}$$

Thus, $41^x \mod 100$ depends on $x \mod 5$. In this case, $x = 42^{18^{9^{5^3}}} \equiv 2^{18^{9^{5^3}}} \mod 5$.
We can find that:

$$\begin{aligned}
2^1 &\equiv 2 \mod 5 \\
2^2 &\equiv 4 \mod 5 \\
2^3 &\equiv 3 \mod 5 \\
2^4 &\equiv 1 \mod 5 \\
2^5 &\equiv 2 \mod 5
\end{aligned}$$

So, $2^y \mod 5$ depends on $y \mod 4$. When $y = 18^{9^{5^3}}$, which is divisible by 4, it must be that $2^y \equiv 2^4 \equiv 1 \mod 5$. Thus, $41^{42^{18^{9^{5^3}}}} \equiv 41^1 \equiv \boxed{\mathbf{41}} \mod 100$

14. The metropolitan area of Nueva-Rosenberg takes the form of a large square grid as shown in the diagram below. Bart wants to get from his home, at point H, to the school, at point S, taking only steps that are either one unit northward or one unit eastward. If the intersections at points A and B are inaccessible, how many distinct paths can Bart take to get to school?



Answer: $\boxed{\textbf{334 paths}}$

Solution: We want to find the number of paths from H to S that don't access A and B. Using the inclusion–exclusion principle, we have that:

$$\begin{aligned}\#\text{ of paths that don't access } A \text{ and } B = &\ \#\text{ of total possible paths} - \#\text{ of paths that access A}\\ &- \#\text{ of paths that access B} + \#\text{ of paths that access A and B}\end{aligned}$$

Ignoring the inaccessibility of A and B, the total number of possible paths from H to S is $\binom{12}{6} = 924$.

The number of paths which access A is $\binom{5}{3} \cdot \binom{7}{3} = 350$ (we consider all the paths which go from H to A, and then all the paths which go from A to S).

The number of paths which access B is $\binom{8}{4} \cdot \binom{4}{2} = 420$ (we consider all the paths which go from H to B, and then all the paths which go from B to S).

The number of paths which access A and B is $\binom{5}{3} \cdot \binom{3}{1} \cdot \binom{4}{2} = 180$ (we consider all the paths which go from H to A, all the paths that go from A to B, and then all the paths which go from B to S).

Thus there are $924 - 350 - 420 + 180 = \boxed{\textbf{334 paths}}$ which don't access A and B.

15. To gain access to the royal palace in the capital of Nuevapolis, Aidin must first beat King Sava III in a game. They start with a pile of n chips, where n is King Sava III's favorite positive integer. With each turn, the players remove 1, 3, or 4 chips from the whole pile, with turns alternating among players and King Sava III going first. Whoever removes the last chip wins. What is the maximal value of $n \leq 111$ such that Aidin can guarantee a win?

Answer: $\boxed{\textbf{107}}$

Solution: Consider the general two-player game described above involving n chips. Let us call n *first-win* if the first player has a winning strategy. Otherwise, call n *second-win*, so that the second player can guarantee a win.

Note that n is second-win if and only if $n-1$, $n-3$, and $n-4$ are all first-win. This is because with n chips, for any first move that the first player makes, the second player will be either starting with $n-1$, $n-3$, or $n-4$ chips.

Also note that if n is second-win, then $n+1$, $n+3$, $n+4$ must all be first-win, as for all of these cases, the first player can make it so that the second player is starting with n chips.

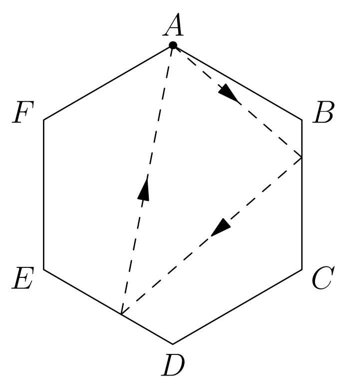
By induction, we show that n is second-win iff $n = 7k+2$ or $n = 7k+7$ for some nonnegative integer k. For the base case, it is easy to check that 1, 3, 4, 5, and 6 are first-win while 2 and 7 are second-win. For the inductive step, suppose that our claim holds for all $k < k'$, where k' is a positive integer. Now, we observe the following using inductive hypothesis:

- Note that $(7k'+1)-1 = (7k'+3)-3 = (7k'+4)-4 = 7k'$ are all second-win by inductive assumption, so $7k'+1$, $7k'+3$, and $7k'+4$ are all first-win.
- We have $(7k'+2)-1 = 7k'+1$, $(7k'+2)-3 = 7(k'-1)+6$, and $(7k'+2)-4 = 7(k'-1)+5$ are all first-win, so $7k'+2$ is second-win.
- Note that $(7k'+5)-3 = (7k'+6)-4 = 7k'+2$ are second-win, so $7k'+5$ and $7k'+6$ are first-win.
- Finally, we have that $(7k'+7)-1 = 7k'+6$, $(7k'+7)-3 = 7k'+4$, and $(7k'+7)-4 = 7k'+3$ are all first-win, so $7k'+7$ is second-win.

Hence $7k'+2$ and $7k'+7$ are second-win while $7k'+1$, $7k'+3$, $7k'+4$, $7k'+5$, and $7k'+6$ are first-win, proving our inductive claim for $k = k'$.

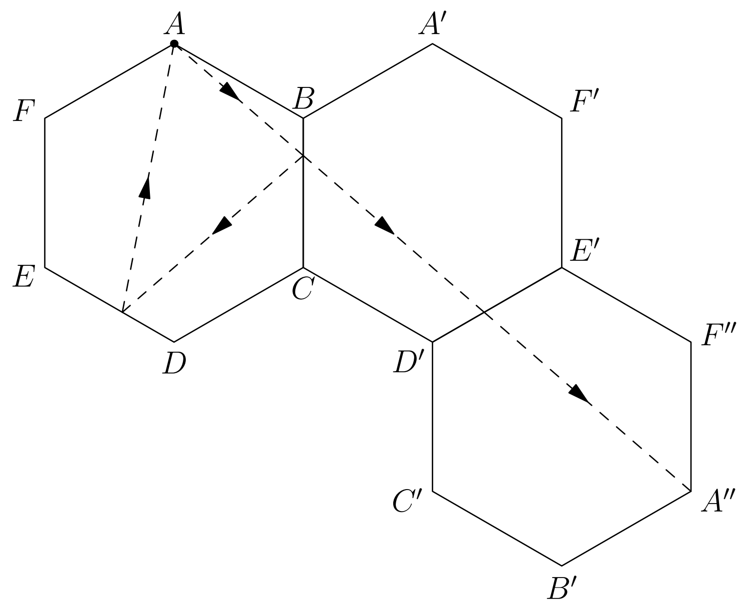
Our original claim thus holds, so Aidin can win iff King Sava III's favorite positive integer n is congruent to 0 or 2 modulo 7. This maximal $n \le 111$ for which this occurs is $n = \boxed{\mathbf{107}}$.

16. Consider a pool table $ABCDEF$ in the shape of a regular hexagon with side length 1. Katie hits a trick shot, hitting a cue ball of negligible size from vertex A, bouncing it off of sides BC and DE and then landing the ball back where it started, as shown by the dotted path below. Assuming the angle of incidence equals the angle of reflection, how far did the ball travel?



Answer: $\boxed{\sqrt{21}}$

Solution:



Since the collisions are perfectly elastic, we may emulate the ball's motion across bounces of the wall by pretending it moves straight through the wall onto a mirrored table as shown in the above diagram.

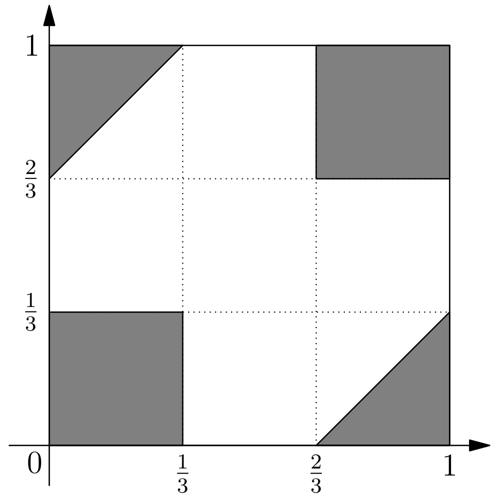
The distance that the ball travels is a straight line from A to A'', so the problem is now finding the length between A and A''.

We set $A = (0, 0)$ and $D = (0, -2)$. We can find from this coordinate projection that $A'' = (2\sqrt{3}, -3)$, meaning that the distance from A to A'' is $\sqrt{(2\sqrt{3})^2 + (-3)^2} = \sqrt{12 + 9} = \boxed{\mathbf{\sqrt{21}}}$.

17. A triangle is formed by choosing three random points on the circumference of a circle. What is the probability that the two smallest angles in the triangle sum to less than $60°$?

Answer: $\boxed{\mathbf{\frac{1}{3}}}$

Solution: WLOG, let one of the points be fixed to the top of the circle. Let us create graph such that the x-axis is the degree angle clockwise of the second point from the top of the circle and the y-axis similarly for the third point. Let each point (x, y) be colored gray if the two smallest angles of the corresponding triangle created sum to less than $60°$, or equivalently, the largest angle is larger than $120°$.



Since all points in the large square are equally probably and the area of the large square is 1, the answer is the area of the gray area. Summing the areas of the four gray areas, we get $\boxed{\mathbf{\frac{1}{3}}}$.

18. Find the number of distinct polynomials P of degree 4 with integer roots such that $P(0) = 2024$ and $P(4) = 0$.

Answer: $\boxed{\mathbf{102}}$

Solution: Since $P(4) = 0$, we write $P = c(x-u)(x-v)(x-w)(x-4)$, where u, v, w, and 4 are the integer roots of P. Then our constraints on P reduce to simply having $cuvw = \frac{P(0)}{4} = \frac{2024}{4} = 506$, with c being an integer. In particular, each polynomial P is determined uniquely by the pair $(c, \{u, v, w\})$, with $\{u, v, w\}$ being a multiset.

Noting that c is a factor of $506 = 2 \cdot 11 \cdot 23$, it follows that either $c = 1$, c is prime, c is semiprime, or $c = 506$.

- If $c = 1$, so that $uvw = 506 = 2 \cdot 11 \cdot 23$, so that $\{|u|, |v|, |w|\}$ is one of $\{2, 11, 23\}$, $\{1, 2, 253\}$, $\{1, 11, 46\}$, $\{1, 22, 23\}$, or $\{1, 1, 506\}$, from which each $\{u, v, w\}$ must be of the form $\{|u|, |v|, |w|\}$, $\{-|u|, -|v|, |w|\}$, $\{|u|, -|v|, -|w|\}$, or $\{-|u|, |v|, -|w|\}$.

  Each of the first 4 options of $\{|u|, |v|, |w|\}$ contains three distinct elements, from which each case gives $4 \cdot 4 = 16$ distinct options for $\{u, v, w\}$ by multiplying $|u|$, $|v|$, and $|w|$ by 1 or -1 are noted above.

  If $\{|u|, |v|, |w|\} = \{1, 1, 506\}$, we note that $\{u, v, w\}$ is either $\{1, 1, 506\}$, $\{-1, -1, 506\}$, and $\{-1, 1, -506\}$, giving 3 additional options.

  Summing yields 19 total options for this case.
- If c is a positive prime, then there exists distinct primes p and q for which $uvw = \frac{506}{c} = pq$. Then $\{|u|, |v|, |w|\}$ can be either $\{1, p, q\}$ or $\{1, 1, pq\}$. Observe that $\{u, v, w\}$ has 4 options in the first case and 3 options in the latter, giving 7 total options.
- If c is a positive semiprime, then $uvw = \frac{506}{c} = p$ for some prime p, so $\{|u|, |v|, |w|\} = \{1, 1, p\}$, which yields 3 options here.
- If $c = 506$, then $uvw = 1$, so $\{u, v, w\}$ is either $\{1, 1, 1\}$ or $\{-1, -1, 1\}$. This gives 2 options.

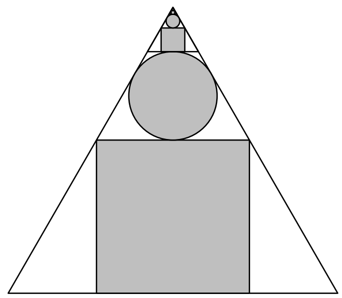
The cases where $c = 1$, c is prime, c is semiprime, or $c = 506$ respectively yield 1, 3, 3, and 1 options for c. Combined with the above, it follows that $(c, \{p, q, r\})$ can take on $1 \cdot 19 + 3 \cdot 7 + 3 \cdot 3 + 1 \cdot 2 = 51$ different values when c is positive.

Observe that the map $f((c, \{u, v, w\})) = (-c, \{-u, -v, -w\})$ sends the $P(x) = c(x-u)(x-v)(x-w)(x-4)$ to $Q(x) = -c(x+u)(x+v)(x+w)(x-4)$, with both P and Q satisfactory. In particular, this bijects the set of satisfactory polynomials with leading coefficient c to the set of satisfactory polynomials with leading coefficient $-c$, so that their counts are the same. In particular, if c is negative, then the pair $(c, \{p, q, r\})$ likewise can take on 51 different possible values.

Summing with the above thus gives $2 \cdot 51 = \boxed{\mathbf{102}}$ total polynomials.

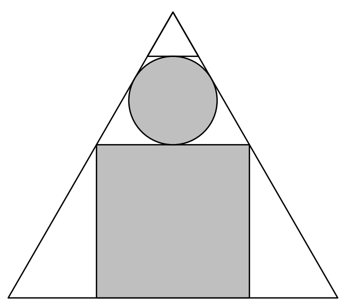
19. 2304

20. There is an equilateral triangle in the plane of side length 2. Alaric inscribes a square in the triangle, splitting the triangle into four regions, one of which is another equilateral triangle. Maxwell inscribes a circle in that new equilateral triangle and draws a line segment tangent to the circle and parallel and disjoint to the bottom side of the new equilateral triangle, thus creating yet another smaller equilateral triangle. Alaric and Maxwell then repeat this process of inscribing a square and a circle on new small equilateral triangles infinitely many times, creating a series of infinitely many squares and circles that get smaller and smaller, as shown in the diagram. The total shaded area, consisting of all squares and circles drawn, can be expressed uniquely as $\frac{a}{b}((c\sqrt{d} - e) + \pi(f\sqrt{g} - h))$, where a, b, c, d, e, f, g, and h are integers, $\gcd(a, b) = 1$, $\gcd(c, e, f, h) = 1$, and d and g are not divisible by any perfect squares. What is $a + b + c + d + e + f + g + h$?



Answer: $\boxed{\mathbf{121}}$

Solution: Let us consider one iteration of the original scenario in a triangle of side length a.



Let the shaded inscribed square have a side length of s. Then the top side of the square divides the large equilateral triangle into a smaller equilateral triangle of side length s and a trapezoid of height s. Matching heights yields $\frac{\sqrt{3}s}{2} + s = \frac{\sqrt{3}a}{2}$ and thus

$$s = \frac{a}{1 + \frac{2}{\sqrt{3}}} = (2\sqrt{3} - 3)a.$$

Now let the shared circle inscribed in the aforementioned equilateral triangle of side length s have radius r. Since an area of any triangle is the product of its semiperimeter and its inradius, with our equilateral triangle having an area of $\frac{\sqrt{3}s^2}{4}$, we thus obtain that

$$r = \frac{\frac{\sqrt{3}s^2}{4}}{\frac{3s}{2}} = \frac{\sqrt{3}s}{6} = \frac{(2 - \sqrt{3})a}{2}.$$

The sum of the areas of the square and circle are thus

$$A(a) = s^2 + \pi r^2 = (2\sqrt{3}-3)^2 a^2 + \frac{\pi(2-\sqrt{3})^2 a^2}{4} = \left(21 - 12\sqrt{3} + \frac{\pi(7-4\sqrt{3})}{4}\right) a^2.$$

The small segment tangent to the circle at its topmost point forms a smaller equilateral triangle. Let its side length be a'. Matching heights gives $\frac{\sqrt{3}a'}{2} = \frac{\sqrt{3}s}{2} - 2r$, so that

$$a' = s - \frac{4\sqrt{3}r}{3} = \frac{(2\sqrt{3}-3)a}{3}.$$

In particular, the ratio between the side lengths of new equilateral triangle formed and the original triangle is exactly

$$\gamma = \frac{a'}{a} = \frac{2\sqrt{3}-3}{3}.$$

To finish, we observe from the above that Alaric and Maxwell's infinite procedure involves the drawing of an infinite series of equilateral triangles with side lengths given by $2, 2\gamma, 2\gamma^2, \ldots$, implying that the total desired shaded area is

$$\begin{aligned} A(2) + A(2\gamma) + A(2\gamma^2) + \ldots &= \left(21 - 12\sqrt{3} + \frac{\pi(7-4\sqrt{3})}{4}\right)(4 + 4\gamma^2 + 4\gamma^4 + \ldots) \\ &= \frac{4\left(21 - 12\sqrt{3} + \frac{\pi(7-4\sqrt{3})}{4}\right)}{1 - \left(\frac{2\sqrt{3}-3}{3}\right)^2} \\ &= \frac{3(36\sqrt{3} - 60 + \pi(3\sqrt{3}-5))}{8}. \end{aligned}$$

Hence $(a,b,c,d,e,f,g,h) = (3,8,36,3,60,3,3,5)$, giving $a+b+c+d+e+f+g+h = \boxed{\mathbf{121}}$.