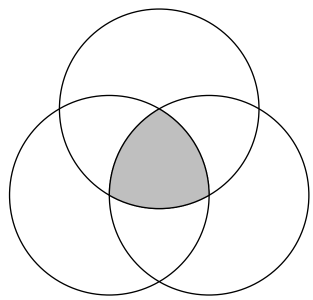
Time Limit: 45 minutes.
Instructions: This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside their appropriate space on the answer sheet will be graded.
No calculators.

1. Bob stands 15 miles west of Alice. Alice starts moving west at 3 mph, and Bob starts moving east at 6 mph. They stop moving once they meet. How far is Bob from his starting position?

2. A perfect square is a number that can be represented as the square of a positive integer. How many positive perfect squares less than 1000 are there?

3. If 5 and 8 are solutions to the quadratic equation $x^2 + ax + b = 0$, what is $b - a$?

4. Consider three circles of radii 2 positioned such that every circle passes through the center of the other two circles. What is the area of the region common to all three circles?



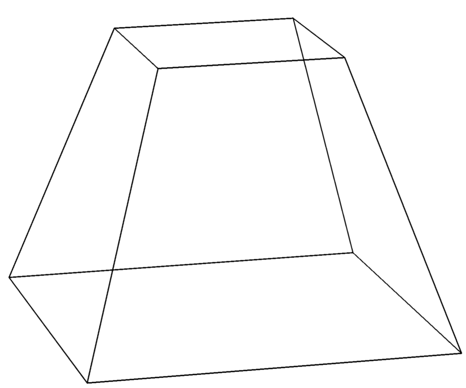
5. A prime number a less than 12 and a positive perfect square b less than 16 are selected uniformly and independently at random. What is the probability that $a + b$ is also a perfect square? Express your answer as a fraction in lowest terms.

6. What is the area of the largest triangle that can be inscribed in a semicircle of radius 5?

7. How many positive integer factors does the number $(1 + 3 + 5 + \cdots + 97 + 99)^2$ have?

8. The Nueva Upper School campus has three floors. During a school day, each student has 4 classes. Kyle always starts off the day with a class on the first floor. Each of his next 3 classes is on a random floor, chosen with equal probability. Before school, his classmate Riley peeks at Kyle's schedule and tells Kyle that none of his back-to-back classes are on the same floor that day. What is the probability that his fourth class is on the first floor? Express your answer as a fraction in the lowest terms.

9. Find the volume of a truncated pyramid with a square base and a side length of 10 cm, a square top with a side length of 5 cm, and a height of 20 cm.

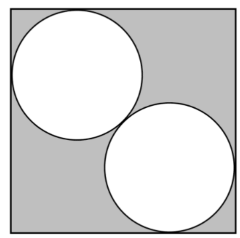


10. Let x be a real number satisfying the equation

$$27 \cdot 4^x + 8 \cdot 9^x = 30 \cdot 6^x.$$

Find the sum of all possible values of x.

11. Two identical circles are tangent to a square of side length 1 and to each other as shown in the diagram. If the area of the shaded region can be expressed as $a - b\pi + c\sqrt{2}\pi$, where a, b, and c are positive integers, compute the value of $a + b + c$.

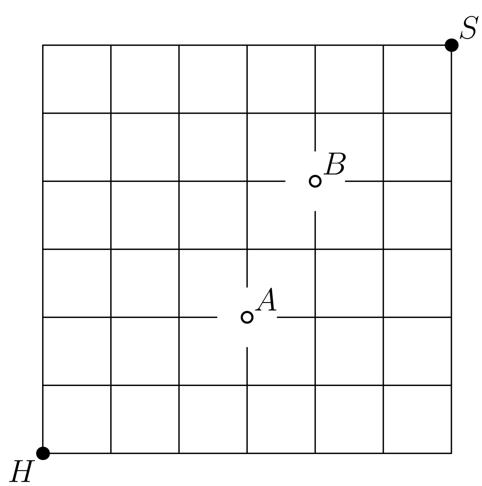


12. Compute the infinite sum

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} = \frac{1^2}{2^1} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots .$$

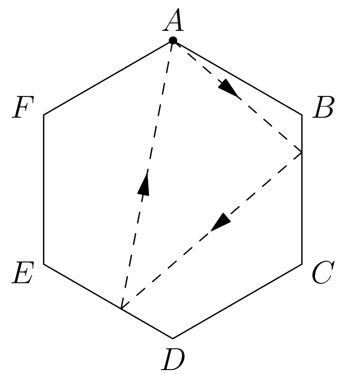
13. What are the last two digits of $41^{42^{18^{9^{5^3}}}}$?

14. The metropolitan area of Nueva-Rosenberg takes the form of a large square grid as shown in the diagram below. Bart wants to get from his home, at point H, to the school, at point S, taking only steps that are either one unit northward or one unit eastward. If the intersections at points A and B are inaccessible, how many distinct paths can Bart take to get to school?



15. To gain access to the royal palace in the capital of Nuevapolis, Aidin must first beat King Sava III in a game. They start with a pile of n chips, where n is King Sava III's favorite positive integer. With each turn, the players remove 1, 3, or 4 chips from the whole pile, with turns alternating among players and King Sava III going first. Whoever removes the last chip wins. What is the maximal value of $n \leq 111$ such that Aidin can guarantee a win?

16. Consider a pool table $ABCDEF$ in the shape of a regular hexagon with side length 1. Katie hits a trick shot, hitting a cue ball of negligible size from vertex A, bouncing it off of sides BC and DE and then landing the ball back where it started, as shown by the dotted path below. Assuming the angle of incidence equals the angle of reflection, how far did the ball travel?



17. A triangle is formed by choosing three random points on the circumference of a circle. What is the probability that the two smallest angles in the triangle sum to less than $60°$?

18. Find the number of distinct polynomials P of degree 4 with integer roots such that $P(0) = 2024$ and $P(4) = 0$.

19. Cain has a Rubik's cube, which is a $3 \times 3 \times 3$ cube made out of 27 unit cubes, where every face on the outside can be rotated by any increment of $90°$. Pomni's house is a corner unit cube of the Rubik's cube. Cain can only rotate, by an increment of $90°, 180°, 270°$, or $360°$, a face that Pomni's house lies on that has not been previous rotated. He wants to perform 6 rotations so that Pomni's is now positioned the corner opposite to its original starting position. In how many ways can this be performed?

20. There is an equilateral triangle in the plane of side length 2. Alaric inscribes a square in the triangle, splitting the triangle into four regions, one of which is another equilateral triangle. Maxwell inscribes a circle in that new equilateral triangle and draws a line segment tangent to the circle and parallel and disjoint to the bottom side of the new equilateral triangle, thus creating yet another smaller equilateral triangle. Alaric and Maxwell then repeat this process of inscribing a square and a circle on new small equilateral triangles infinitely many times, creating a series of infinitely many squares and circles that get smaller and smaller, as shown in the diagram. The total shaded area, consisting of all squares and circles drawn, can be expressed uniquely as $\frac{a}{b}((c\sqrt{d} - e) + \pi(f\sqrt{g} - h))$, where a, b, c, d, e, f, g, and h are integers, $\gcd(a, b) = 1$, $\gcd(c, e, f, h) = 1$, and d and g are not divisible by any perfect squares. What is $a + b + c + d + e + f + g + h$?