1. What is $\frac{2^4+2^4}{2^{-4}+2^{-4}}$?

   Answer: $\boxed{\textbf{256}}$

   Solution:

   We calculate $2^4 + 2^4 = 16 + 16 = 32$ and $2^{-4} + 2^{-4} = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$. So, we have that $\frac{2^4+2^4}{2^{-4}+2^{-4}} = \frac{32}{\frac{1}{8}} = \boxed{\textbf{256}}$.

2. What is the sum of the solutions of $|x^2 - 9x + 17| = 3$?

   Answer: $\boxed{\textbf{18}}$

   Solution:

   We have two cases:

   Case 1: $x^2 - 9x + 17 = 3$

   We have that $x^2 - 9x + 14 = 0$. Factoring this gives us $(x - 2)(x - 7) = 0$, which has solutions $x = 2, 7$.

   Case 2: $x^2 - 9x + 17 = -3$

   We have that $x^2 - 9x + 20 = 0$. Factoring this gives us $(x - 4)(x - 5) = 0$, which has solutions $x = 4, 5$.

   After checking that all of the solutions actually satisfy the equation, we can calculate the sum of the solutions of $|x^2 - 9x + 17| = 3$ to be $2 + 7 + 4 + 5 = \boxed{\textbf{18}}$.

3. How many different ways can Ryan choose exactly 6 balls from a bag containing 3 red, 3 blue, and 5 green, where balls of each color are indistinguishable?

   Answer: $\boxed{\textbf{15}}$

   Solution:

   Since the balls are indistinguishable, each different way that Ryan can choose the 6 balls can be reduced to how many red, blue, and green balls he took.

   From here, we can use casework:

   Case 1: Ryan takes 0 red balls.

   We now need 6 more balls, and this can be split into 1 blue and 5 green, 2 blue and 4 green, and 3 blue and 3 green. Thus we have 3 ways from this case.

   Case 1: Ryan takes 1 red ball.

   We now need 5 more balls, and this can be split into 0 blue and 5 green, 1 blue and 4 green, 2 blue and 3 green, and 3 blue and 2 green. Thus we have 4 ways from this case.

   Case 2: Ryan takes 2 red balls.

   We now need 4 more balls, and this can be split into 0 blue and 4 green, 1 blue and 3 green, 2 blue and 2 green, and 3 blue and 1 green. Thus we have 4 ways from this case.

   Case 3: Ryan takes 3 red balls.

   We now need 3 more balls, and this can be split into 0 blue and 3 green, 1 blue and 2 green, 2 blue and 1 green, and 3 blue and 0 green. Thus we have 4 ways from this case.

   Adding them up, we get $3 + 4 + 4 + 4 = \boxed{\textbf{15}}$.

4. Compute $3 + 10 + 17 - 24 + 31 + 38 + 45 - 52 + \cdots + 311 + 318 + 325 - 332$.

   Answer: $\boxed{\textbf{3768}}$

   Solution:

   We can notice that the terms follow the pattern of

$$(3 + 7n) + (3 + 7(n + 1)) + (3 + 7(n + 2)) - (3 + 7(n + 3))$$

for $4|n$ (meaning n divisible by 4). By rearranging, we can see that the terms are equal to $6 + 7(2n)$. Since the first term has an n value of 0 and the last term has an n value of $\frac{311-3}{7} = 44$, our sum becomes

$$6 + 0 + 6 + 7(2 \cdot 4) + \cdots + 6 + 7(2 \cdot 44).$$

Since there are 12 terms in the sequence, we can simplify:

$$6 \cdot 12 + 14 \cdot 4(0 + 1 + \cdots + 11) = 72 + 56\frac{11(11+1)}{2} = 72 + 56 \cdot 66 = \boxed{\mathbf{3768}}.$$

5. In a pile of 70 M&Ms, there are 28 blue M&Ms. Jill chooses 3 M&Ms at random one at a time. Let $\frac{a}{b}$ be the probability that she chose 3 blue M&Ms, where a and b are relatively prime integers. What is a?

Answer: $\boxed{\mathbf{117}}$

Solution:

To get three blue M&Ms, the first M&M, the second M&M, and the third M&M each have to be blue. The probability of the first M&M being blue is $\frac{28}{70}$. Given that the first M&M is blue, the probability of the second M&M being blue is $\frac{27}{69}$. Given that the first two M&Ms are blue, the probability of the third M&M being blue is $\frac{26}{68}$. Thus, the probability of all three M&Ms being blue is $\frac{28}{70} \cdot \frac{27}{69} \cdot \frac{26}{68} = \frac{117}{1955}$, so the answer is $\boxed{\mathbf{117}}$.

6. Find the integer closest to $(3 + 2\sqrt{2})^4$.

Answer: $\boxed{\mathbf{1154}}$

Solution:

We compute $(3 + 2\sqrt{2})^4$ to be:

$$\begin{aligned}
(3 + 2\sqrt{2})^4 &= ((3 + 2\sqrt{2})^2)^2 \\
&= (9 + 12\sqrt{2} + 8)^2 \\
&= (17 + 12\sqrt{2})^2 \\
&= 289 + 408\sqrt{2} + 288 \\
&= 577 + 408\sqrt{2}
\end{aligned}$$

Note that $(3 + 2\sqrt{2})^4 + (3 - 2\sqrt{2})^4 = 577 + 408\sqrt{2} + 577 - 408\sqrt{2} = 1154$. Also notice that $0 < 3 - 2\sqrt{2} < \frac{1}{2}$, so $0 < (3 - 2\sqrt{2})^4 < \frac{1}{2}$. Thus, the integer closest to $(3 + 2\sqrt{2})^4$ is $\boxed{\mathbf{1154}}$.

7. How many positive integers less than or equal to 1000 are divisible by exactly one of 2, 3, or 5?

Answer: $\boxed{\mathbf{468}}$

Solution:

Using the principle of inclusion-exclusion, we calculate this number to be:

$$\begin{aligned}
N = {} & [\text{\# of integers divisible by 2}] + [\text{\# of integers divisible by 3}] + [\text{\# of integers divisible by 5}] \\
& - 2\left([\text{\# of integers divisible by 6}] + [\text{\# of integers divisible by 10}] + [\text{\# of integers divisible by 15}]\right) \\
& + 3\left([\text{\# of integers divisible by 30}]\right) \\
& = 500 + 333 + 200 - 2(166 + 100 + 66) + 3(33) \\
& = \boxed{\mathbf{468}}
\end{aligned}$$

8. Let x and y be real numbers that satisfy $x + y = 4$ and $xy = 2$. What is the value of $x^4 + y^4$?

   Answer: $\boxed{\textbf{136}}$

   Solution:

   We first find $x^2 + y^2$:

$$\begin{aligned} x^2 + y^2 &= x^2 + 2xy + y^2 - 2xy \\ &= (x + y)^2 - 2xy \\ &= 4^2 - 2 \cdot 2 \\ &= 12 \end{aligned}$$

   Then, we find $x^4 + y^4$:

$$\begin{aligned} x^4 + y^4 &= x^4 + 2x^2y^2 + y^4 - 2x^2y^2 \\ &= (x^2 + y^2)^2 - 2(xy)^2 \\ &= 12^2 - 2 \cdot 2^2 \\ &= 144 - 8 \\ &= \boxed{\textbf{136}} \end{aligned}$$

9. Find the number of subsets of $\{1, 2, 3, 4, 5\}$ whose elements add up to 8.

   Answer: $\boxed{\textbf{3}}$

   Solution:

   The subsets whose elements add up to 8 are $\{1, 3, 4\}, \{1, 2, 5\}, \{3, 5\}$. So, there are $\boxed{\textbf{3}}$ valid subsets.

10. The sides of a triangle have lengths 10, 24, and 26. Let R be the radius of the circumcircle of this triangle and r be the radius of the incircle of this triangle. Find $R - r$.

    Answer: $\boxed{\textbf{9}}$

    Solution:

    Let $\triangle ABC$ denote the triangle of side lengths $a = 10$, $b = 24$, and $c = 26$. Note that $\triangle ABC$ is a right triangle as $a^2 + b^2 = c^2$. Thus it is immediate that $R = \frac{c}{2} = 13$. We also have $\frac{ab}{2} = [\triangle ABC] = rs$ where $s = \frac{a+b+c}{2}$, so

$$r = \frac{ab}{a + b + c} = \frac{10 \cdot 24}{10 + 24 + 26} = 4$$

    and $R - r = 13 = 4 = \boxed{\textbf{9}}$.

11. A regular hexagon is inscribed in a circle of radius 1. Find the sum of the squares of all sides and diagonals of this hexagon.

    Answer: $\boxed{\textbf{36}}$

    Solution:

    Each of the 6 sides of the hexagon are of length 1. Furthermore, each of the 6 non-diameter diagonals have length $\sqrt{3}$, and each of the 3 diameters have length 2. We check that all sides are accounted for as $6 + 6 + 3 = 15 = \binom{6}{2}$. Thus the answer is $6 \cdot 1^2 + 6(\sqrt{3})^2 + 3 \cdot 2^2 = \boxed{\textbf{36}}$.

12. Let $f(x) = x^3 - 24x^2 + 188x - 480$. Find the sum of all possible $af(a)$ such that $f(a)$ is a positive, prime number.

    Answer: $\boxed{\textbf{21}}$

Solution:

Note that

$$f(x) = x^3 - 24x^2 + 188x - 480 = (x-6)(x-8)(x-10).$$

Hence, if $f(a)$ is prime, then $x = 7$ or $x = 9$. Since $f(7) = -f(9) = 3$, the answer is $7f(7) = \boxed{\mathbf{21}}$.

13. Evaluate $\sqrt{100 \cdot 102 \cdot 104 \cdot 106 + 16}$.

Answer: $\boxed{\mathbf{10604}}$

Solution:

Observe that

$$\begin{aligned}\sqrt{2x(2x+2)(2x+4)(2x+6)+16} &= 4\sqrt{x(x+1)(x+2)(x+3)+1} \\ &= 4\sqrt{x^4+6x^3+11x^2+6x+1} \\ &= 4(x^2+3x+1).\end{aligned}$$

Substituting in $x = 50$ yields an answer of $\boxed{\mathbf{10604}}$.

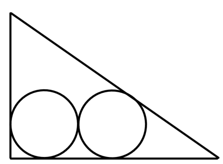
14. Connor is a member of the Nueva Math Club, which meets once every 7 days, Student Council, which meets once every 3 days, and Physics Club, which meets once every 5 days. Assume that Math Club, Student Council, and Physics Club all meet today. In 315 days, how many days will Connor not attend a meeting?

Answer: $\boxed{\mathbf{144}}$

Solution:

Since $315 = 3 \cdot (3 \cdot 5 \cdot 7)$, the answer is just $3\phi(105) = 3 \cdot (2 \cdot 4 \cdot 6) = \boxed{\mathbf{144}}$.

15. Let ABC be a triangle such that $\overline{AB} = 3$, $\overline{BC} = 4$, and $\overline{AC} = 5$. Circles r_1 and r_2, each with radius r, are drawn such that r_1 and r_2 are internally tangent to $\overline{BC}$, r_1 is internally tangent to $\overline{AB}$, and r_2 is internally tangent to $\overline{AC}$. If r can be written in the form $\frac{a}{b}$, where a and b are relatively prime integers, find $a + b$.



Answer: $\boxed{\mathbf{5}}$

Solution:

Suppose that we embed this triangle on the coordinate plane, where $B = (0,0)$, $A = (0,3)$, and $C = (4,0)$. Let the radius of each circle have length r. Then, the center of the right circle has coordinate $(3r, r)$. The point on $\overline{AC}$ that this circle is tangent to, which we note as point D, is at $(3r + \frac{3}{5}r, r + \frac{4}{5}r) = (\frac{18}{5}r, \frac{9}{5}r)$.

We note that $\overline{AC}$ has equation $3x + 4y = 12$. Substituting $x = \frac{18}{5}r$ and $y = \frac{9}{5}r$ into this equation, we get:

$$\begin{aligned}3\left(\frac{18}{5}r\right) + 4\left(\frac{9}{5}r\right) &= 12 \\ \frac{54}{5}r + \frac{36}{5}4 &= 12 \\ \frac{90}{5}r &= 12 \\ 18r &= 12 \\ r &= \frac{2}{3}\end{aligned}$$

So, the answer to this problem is $2 + 3 = \boxed{5}$

16. Let n be a positive integer greater than 1 and less than 100. For how many values of n is $7^0 + 7^1 + \ldots + 7^n$ divisible by 100?

Answer: $\boxed{\mathbf{25}}$

Solution:

We have:

$$\begin{aligned} 7^0 + \ldots + 7^n &\equiv 0 \mod 100 \\ \frac{7^{n+1} - 1}{6} &\equiv 0 \mod 100 \\ 7^{n+1} - 1 &\equiv 0 \mod 600 \\ 7^{n+1} &\equiv 1 \mod 600 \end{aligned}$$

Note that $7^4 = 2401 \equiv 1 \mod 600$. Thus, $4|n+1$. So, we find that there are $\boxed{\mathbf{25}}$ possible values of n.

17. Let a, b, c, and d be positive integers with $a + 64c = 8b + 512d$, $16a + 36c = 24b + 54d$, and $a + 9c = 3b + 27d$. Find the least possible value of $a + b + c + d$.

Answer: $\boxed{\mathbf{180}}$

Solution:

Denote the equations expressed in the problem as as equations (1), (2), and (3).

We use equation (1) to get that $a = 8b - 64c + 512d$. We substitute this into equation (3):

$$\begin{aligned} 8b - 64c + 512d + 9c &= 3b + 27d \\ 5b &= 55c - 485d \\ b &= 11c - 97d \end{aligned}$$

We also substitute $a = 8b - 64c + 512d$ into equation (2):

$$\begin{aligned} 16(8b - 64c + 512d) + 36c &= 24b + 54d \\ 128b - 1024c + 8192d + 36c &= 24b + 54d \\ 104b &= 988c - 8138d \\ b &= \frac{19}{2}c - \frac{313}{4}d \end{aligned}$$

We set these two expressions for b equal to each other:

$$\begin{aligned} 11c - 97d &= \frac{19}{2}c - \frac{313}{4}d \\ \frac{3}{2}c &= \frac{75}{4}d \\ c &= \frac{25}{2}d \end{aligned}$$

We substitute this value of c into $b = 11c - 97d$:

$$\begin{aligned} b &= 11\left(\frac{25}{2}d\right) - 97d \\ &= \frac{275}{2}d - \frac{194}{2}d \\ &= \frac{81}{2}d \end{aligned}$$

If we substitute these values of b and c into $a = 8b - 64c + 512d$, we get:

$$\begin{aligned} a &= 8\left(\frac{81}{2}d\right) - 64\left(\frac{25}{2}d\right) + 512d \\ &= 324d - 800d + 512d \\ &= 36d \end{aligned}$$

So, we have that $a = 36d$, $b = \frac{81}{2}d$, and $c = \frac{25}{2}d$. The smallest value of d which makes a, b, c, and d all integers is $d = 2$. So, we have that:

$$\begin{aligned} a + b + c + d &= 72 + 81 + 25 + 2 \\ &= \boxed{\mathbf{180}} \end{aligned}$$

18. Jack and Bob are preparing for a cross country meet. To do this, they will practice running on an empty road which has an infinite amount of light poles, each spaced 10 meters apart, and numbered $0, 1, 2, 3, 4, 5, \ldots$ Jack and Bob start at lightpole a, and they begin running to lightpole 47, with $a < 47$. The ratio of Jack to Bob's speed is $5 : 6$, and whenever one of them encounters lightpole a or 47, they immediately start running in the opposite direction, never stopping. In addition, Jack and Bob always maintain their speeds, never slowing down or speeding up. After they take off, they cross each other for the 1st and 2nd time at lightpoles p and 20, respectively. Find the value of p.

Answer: $\boxed{\mathbf{44}}$

Solution:

Suppose Jack and Bob start at position $x = 0$ and pole 47 is at position $x = d$. When Bob gets to position $x = d$, Jack will be at position $x = \frac{5}{6}d$. So, they will meet for the first time at $x = \frac{5}{6}d + \frac{5}{66}d = \frac{10}{11}d$.

When Jack reaches pole 47, Bob will be at position $x = \frac{10}{11}d - \frac{6}{5} \cdot \frac{1}{11}d = \frac{4}{5}d$. When Bob reaches pole a, Jack will be at position $x = d - \frac{5}{6} \cdot \frac{4}{5}d = \frac{1}{3}d$. So, they will meet for the second time at $x = \frac{6}{11} \cdot \frac{1}{3}d = \frac{2}{11}d$.

So, we have that pole 20 is at position $x = \frac{2}{11}d$. Knowing that pole a is at position $x = 0$ and pole 47 is at position $x = d$, we have that $a = 14$. We can then find that $p = 14 + \frac{10}{11}(47 - 14) = \boxed{\mathbf{44}}$.

19. The Nueva Pizzeria offers 4 toppings: pepperoni, sausage, onions, and mushrooms. Hans orders a large circular pizza which has 8 slices, in which each slice is a sector of the pizza. However, he insists that every slice must have exactly 1 topping and any adjacent slices must have different toppings. How many different pizzas can be ordered? Rotations and reflections are considered distinct.

Answer: $\boxed{\mathbf{6564}}$

Solution:

We want to find the number of possible sequences $a_1, a_2, \ldots, a_7, a_n$ such that $a_i \in \{1, 2, 3, 4\}$ for all $1 \leq i \leq n$, and two adjacent values are equal to each other, and $a_1 \neq a_n$.

Let $f(n)$ equal the number of sequences which satisfy the first two conditions, but not the latter, and let $g(n)$ equal the number of sequences which satisfy all three conditions. We'd like to find $g(8)$. We can define $f(n)$ and $g(n)$ recursively as such:

$$\begin{aligned} f(n) &= g(n-1) \\ g(n) &= 3f(n-1) + 2g(n-1) \\ &= 3g(n-2) + 2g(n-1) \end{aligned}$$

So, we find $g(8)$:

$$\begin{aligned} g(1) &= 0 \\ g(2) &= 4 \cdot 3 = 12 \\ g(3) &= 3(0) + 2(12) = 24 \\ g(4) &= 3(12) + 2(24) = 84 \\ g(5) &= 3(24) + 2(84) = 240 \\ g(6) &= 3(84) + 2(240) = 732 \\ g(7) &= 3(240) + 2(732) = 2184 \\ g(8) &= 3(732) + 2(2184) = 6564 \end{aligned}$$

So, there are $\boxed{\textbf{6564}}$ possible pizzas.

20. Let $ABCD$ be a parallelogram with angle $\angle A < 90^\circ$, let E be the center of a circle tangent to $\overline{AB}$, $\overline{BC}$, and $\overline{DA}$, and let F be a point on the circumference of the circle centered at E. The distances from F to lines $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are 5, 14, 15, and 6, respectively. The perimeter of $ABCD$ can be written as $\frac{a\sqrt{b}(\sqrt{c}-d)}{e}$, where a, b, c, d, and e are positive integers, a and e do not share any common factors greater than 1, and b and c are not divisible by any perfect square greater than 1. Find $a + b + c + d + e$.

Answer: $\boxed{\textbf{821}}$

Solution:

Note that distance between $\overline{AB}$ and $\overline{CD}$ is $5 + 15 = 20$, and the distance between $\overline{BC}$ and $\overline{DA}$ is $14 + 6 = 20$. So, parallelogram $ABCD$ is a rhombus.

Let $A = (0, 0)$, $B = (2\sqrt{b^2 - 100}, 20)$, $C = (2b, 0)$, $D = (2(\sqrt{b^2 - 100} + b), 20)$. Note that the perimeter of the parallelogram is $8b$.

We have that $E = (\sqrt{b^2 - 100} + b, 10)$. Thus, the circle centered at E has equation $(x - (\sqrt{b^2 - 100} + b))^2 + (y - 10)^2 = 100$.

We set $F = (x_0, 6)$, since the distance from F to $\overline{CD}$ is 6. The line $\overline{AB}$ has equation $y\sqrt{b^2 - 100} - 10x = 0$. We use the formula for the distance from F to $\overline{AB}$ to get:

$$\begin{aligned} \frac{|6\sqrt{b^2 - 100} - 10x_0|}{b} &= 5 \\ 10x_0 - 6\sqrt{b^2 - 100} &= 5b \\ x_0 &= \frac{5b + 6\sqrt{b^2 - 100}}{10} \end{aligned}$$

Substituting $x = x_0$ and $y = 6$ in the equation for the circle centered at E, we have:

$$\begin{aligned}
\left(\frac{5b+6\sqrt{b^2-100}}{10}-(\sqrt{b^2-100}+b)\right)^2+(6-10)^2 &= 100\\
\frac{5b+4\sqrt{b^2-100}}{10} &= 2\sqrt{21}\\
4\sqrt{b^2-100} &= 20\sqrt{21}-5b\\
16b^2-1600 &= 25b^2-200\sqrt{21}b+8400\\
9b^2-200\sqrt{21}b+10000 &= 0\\
b &= \frac{200\sqrt{21}\pm\sqrt{840000-360000}}{18}\\
&= \frac{100\sqrt{21}\pm 200\sqrt{3}}{9}\\
&= \frac{100\sqrt{3}(\sqrt{7}+2)}{9}
\end{aligned}$$

So, the perimeter of the parallelogram is $8b = \frac{800\sqrt{3}(\sqrt{7}+2)}{9}$. Therefore, the answer to this problem is $800+3+7+2+9=\boxed{\mathbf{821}}$.