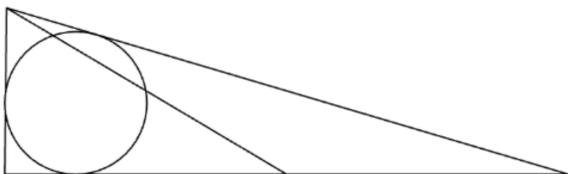


**Time Limit:** 30 minutes.

**Instructions:** For this test, work in teams of up to four to solve 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside their appropriate space on your team answer sheet will be graded.

**No calculators.**

1. Let  $ABC$  be a triangle where  $\overline{AB} = 7$  and  $\overline{AC} = 13$ . Let  $D$ ,  $E$ , and  $F$  all be on  $\overline{BC}$  such that  $\overline{AD}$  is an altitude from  $A$ ,  $\overline{AE}$  is an angle bisector of  $\angle BAC$ , and  $\overline{AF}$  is a median from  $A$ . If  $\overline{DE} = \overline{EF}$ , find  $\overline{BC}^2$
2. Suppose Bill has jars of 4, 7, and 10 jelly beans. Assuming that he has enough jars, what is the largest number of jelly beans that Bill cannot make using these jars?
3. For all subsets of  $\{1, \dots, 100\}$ , the expected number of consecutive values in the set (for example,  $\{2, 3, 4\}$  has 2 consecutive values) is  $\frac{a}{b}$ , where  $a$  and  $b$  are relatively prime. Find  $a + b$ .
4. The solution of the equation  $4^{x+8} = 9^x$  can be expressed in the form  $\log_a(4^4)$ . What is  $a$ ? Express your answer in terms of a fraction in simplest form
5. Given three positive numbers  $a$ ,  $b$ ,  $c$ , the minimum possible value of  $\frac{a^2}{(b+c)^2+a^2} + \frac{b^2}{(c+a)^2+b^2} + \frac{c^2}{(a+b)^2+c^2}$  is  $\frac{x}{y}$ , where  $x$  and  $y$  are relatively prime positive integers. Find  $x + y$ .
6. Let  $\triangle ABC$  be a triangle with  $\overline{AB} = 26$ ,  $\overline{AC} = 17$ , and  $\overline{BC} = 25$ . Define points  $H_1, H_2, \dots$  as follows: Let  $H_1$  be the altitude from  $B$  to  $\overline{AC}$  (the point on  $\overline{AC}$  that angle  $\angle AH_1B$  is right), let  $H_2$  be the altitude from  $H_1$  to  $\overline{AB}$ , and let  $H_{n+2}$  be the altitude from  $H_{n+1}$  to  $\overline{AH}_n$  for  $n \geq 2$ . Find the infinite sum  $\overline{BH}_1 + \overline{H_1H_2} + \overline{H_2H_3} + \dots$
7. Let  $a_0, a_1, a_2, \dots$  be an arithmetic sequence, and let  $g_0, g_1, g_2, \dots$  be a geometric sequence. Given that  $a_0 = 108$ ,  $g_0 = 2$ ,  $a_{10} = g_6$ , and  $a_2 = 7g_3$ , find the sum of all possible values of  $a_1 + g_1$ .
8.  $\triangle ABC$  is a triangle with longest side  $\overline{AC}$  and  $BC = 24$ , and point  $D$  lies on  $BC$  such that  $\overline{BD} = \overline{CD}$ .  $\overline{AD}$  intersects the incircle of  $\triangle ABC$  at 2 points:  $E$  and  $F$ , with  $E$  closer to  $A$  than  $F$ , and the ratio of  $\overline{AE} : \overline{EF} : \overline{FD}$  is  $1 : 2 : 3$ . If the radius of the incircle of  $\triangle ABC$  can be written as  $\frac{a\sqrt{b}}{c}$  where  $a$ ,  $b$ , and  $c$  are positive integers,  $b$  is not divisible by any perfect square greater than 1, and  $a$  and  $c$  are relatively prime, find  $a + b + c$ .



9. Find the amount of nonempty subsets of  $\{1, 2, 3, \dots, 20\}$  such that any two (not necessarily distinct) elements differ by a factor of at most 2, and no two elements are consecutive. For example, the subsets  $\{3, 5\}$ ,  $\{5, 8, 10\}$ , are valid subsets, while the subsets  $\{1, 3\}$ ,  $\{3, 9, 11, 16, 19\}$ , and  $\{6, 7, 11\}$  are not valid.
10. Let  $\sqrt{64 - x^2} - \sqrt{49 - x^2} = 3$ . What is the value of  $\sqrt{64 - x^2} + \sqrt{49 - x^2}$ ?