Time Limit: 45 minutes.
Instructions: This test contains 20 short answer questions. All answers must be expressed as integers unless specified otherwise. Only answers written inside their appropriate space on the answer sheet will be graded.
No calculators.

1. Abby has 20 fruits. Bob takes 2 fruits from Abby, and Catherine takes 6 fruits from Abby. How many fruits does Abby have now?

2. Eddy doubles a number and subtracts the result from 17 to get 5. What was his original number?

3. Find the last 2 digits of 11^{11}.

4. Ryan has 2 apples, Harrison has 5 oranges, and Connor has 10 plums. Connor gives Ryan half of his plums, and Harrison gives Ryan 2 oranges. How many fruits does Ryan have now?

5. Eddy needs to paint 50 houses, numbered 1 to 50. Eddy paints the walls of every even-numbered house white, and paints the roofs of every house whose number is divisible by 3 red. How many houses now have both white walls and red roofs? He doesn't paint any other houses these colors.

6. What are the last two digits of 1 + 12 + 123 + 1234?

7. In the city of New San Mateo, a baby is born every 3 hours, and two people die every day. How many people will be added to the population of New San Mateo during the entire month of June? (June has 30 days).

8. For two numbers a and b, let $a@b = a + 2b$, and $a\#b = b - a$. Find the value of $(5\#2)@3$.

9. Elbert is making shapes out of equilateral triangular tiles by joining them at their edges. How many different connected shapes can he make with 4 tiles? (Shapes that can be matched using rotation are considered the same, but those that require flipping upside-down or left-right to match are considered different.)

10. A, B, and C are distinct digits between 1 and 9, inclusive, such that $A \cdot B = 24$ and $B + C = 17$. What is $A + B + C$?

11. Consider the sequence $\frac{1000}{1}, \frac{1000}{3}, \frac{1000}{5}, \frac{1000}{7}, \ldots$. What is the closest integer to the 18th term of the sequence?

12. The perimeter of a rectangle with integer side lengths is 38. What is the absolute difference between the largest and smallest possible areas of the rectangle?

13. Let X, Y, Z be equilateral triangles. Suppose that Y has twice the perimeter of X, and Z has triple the area of Y. What is the ratio between the area of the Z to the area of X, expressed as an integer?

14. Alex is walking in a field of flowers placed in the coordinate grid. At every point with integer coordinates, there is either a red flower or a blue flower, with an equal probability. When Alex encounters a red flower, he walks one unit in the +x direction, and when he encounters a blue flower, he moves one unit in the +y direction. Given that he starts at a blue flower, at the moment when Alex has moved 67 units in the +y direction from his initial position, what is the expected number of units he has walked in the +x from his initial position?

15. In $\triangle ABC$ with $AB = 9$, $BC = 40$, and $AC = 41$, point D is on BC such that $BD = 28$, and point E is the midpoint of AD. If line CE intersects AB at point F, then BF can be represented in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

16. Pencill & Banker, a Magic Show Duo, are preparing for their next big magic trick, which involves a staple gun. Banker will press the trigger of the staple gun 7 times. Every time, it will either fire an actual staple, or a blank. To maximize awe in the audience, they want the order of staples and blanks to be entertaining. An ordering of 7 staples and blanks is entertaining if there are at least 3 staples and at least 3 blanks, and there are never 3 blanks in a row or 3 staples in a row. How many ways can they entertain the audience?

17. Ryan colors the cells of the following 3 by 3 grid either black and white, such that the number next to each row and column corresponds to the amount of black squares. How many ways can he color the grid?

	1	1	2
1			
2			
1			

18. Let $S(n)$ denote the sum of the digits of a positive integer n. If the digits of positive integer m which consists of some amounts of 1's, 5's, 8's, along with 52 2's and 36 7's satisfies $S(m) = 700$, $S(2m) = 635$, and $S(3m) = 840$, how many digits long is m?

19. Find the smallest odd prime p that divides $3^{11} + 1$.

20. There exists a polynomial $f(x)$ of degree 100 such that for all n in $\{0, 1, ..., 100\}$, $f(n) = \frac{n}{n+2}$. If $f(101) = \frac{p}{q}$, where p and q are relatively prime, find $p + q$.